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THE SOLUTION OF NONLINEAR BOUNDARY VALUE PROBLEM IN AN UNBOUNDED DOMAIN

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Abstract. In this paper we consider the problem of finding the speed of solitary waves in saturated porous media. The problem of propagation of solitary waves is reduced to the solution of the boundary value problem in unbounded domain. For such problems boundary conditions are specified in infinity. In the first approximation for the dispersion equation the velocity of the stationary running linear waves is obtained, in the second approximation solution of the nonlinear evolution equations is derived. The first equation is solved as a system of linear algebraic equations. To solve the second equation quasi-uniform grid is constructed, covering the spatial domain and difference schemes are received for the numerical solution of the equations describing the propagation of nonlinear waves.

Keywords: Solutary waves, nonlinear equations, unbounded domain, a quasi-uniform grid, difference schemes.

AMS Subject Classification: 35Q51, 37K40.

1. Introduction

The mathematically posed problem for one-dimensional and isothermic case is led to the solution of the mass and momentum conservation equation [1]

$$\frac{\partial(\alpha_i\rho_i)}{\partial t}+\frac{\partial(\alpha_i\rho_i\upsilon_i)}{\partial x}=0\,,\alpha_1+\alpha_2=1, \tag{1}$$

$$\frac{\partial(\alpha_i\rho_i\upsilon_i)}{\partial t}+\frac{\partial(\alpha_i\rho_i\upsilon_i\upsilon_i)}{\partial x}=\delta_{1i}\frac{\partial\sigma}{\partial x}+\alpha_i\frac{\partial P}{\partial x}-(-1)^i R_{12}, \tag{2}$$

We take the connection between stress and deformation of solid phase in the form [1]

$$\left(b_0+\sum_{l=1}^{m}b_l\frac{D^l}{Dt^l}\right)(\sigma+\gamma P)=\left(a_0+\sum_{l=1}^{n}a_l\frac{D^l}{Dt^l}\right)e_1, \tag{3}$$

The system of equations (1)-(3) is completed by the thermodynamic equation of phase states

$$\rho_1=\rho_1(\sigma_1,P),\qquad \rho_2=\rho_2(P), \tag{4}$$

The parameters of solid and fluid phases are denoted here by indices 1 and 2, respectively. The portion of volume occupied by solid phase is equal to α_1, fluid phase – to α_2, the density of substance in different phases is equal to ρ_1 and ρ_2, rates are υ_1, υ_2, $\sigma = \alpha_1(\Gamma - P)$, P is pressure in the fluid phase, Γ is the true stress in the solid phase, δ_{ij} - is a unit tensor, $\alpha_1^{(0)}$ is an initial value of α_1; $\gamma = \beta_1 K$; β_1 and K are the coefficients of isothermic compressibility of solid particles and all solid phase as a whole.

The constant coefficients $b_0, b_1, ..., b_m; a_0, a_1, ..., a_n$ are defined from the concrete elastico-viscous models. We can give the force of interfacial resistance in the form [1]

$$R_{12} = (\upsilon_2 - \upsilon_1) f\left(\left|\bar{\upsilon}_2 - \bar{\upsilon}_1\right|\right) = K_\nu(\upsilon_2 - \upsilon_1) + K_\nu b(\upsilon_2 - \upsilon_1)\left|\bar{\upsilon}_2 - \bar{\upsilon}_1\right| . \tag{5}$$

The domains of correctness of law (5) and linear connection (when $b = 0$) are defined by the Reynolds inner number $\mathrm{Re} = ul\rho_2 / \nu = (\rho_2 / \nu)\left|\bar{\upsilon}_2 - \bar{\upsilon}_1\right|\sqrt{k / \alpha_2}$ and its critical value Re_{kr} after that the linear connection get broken, where ν is the dynamic fluid viscosity, k is a permeability coefficient of the porous medium.

Formula (5) can be represented in a more convenient form

$$R_{12} = (\varphi(\mathrm{Re}))(\nu\alpha_2 / k)\left|\bar{\upsilon}_2 - \bar{\upsilon}_1\right| .$$

The values of the function $\varphi(\mathrm{Re})$ for different porous media have been given in [1]. The equations (1)-(4) are completed by the following kinematic relation

$$\frac{\partial e_1}{\partial t} + \frac{\partial e_1 \upsilon_1}{\partial x} = \frac{\partial \upsilon_1}{\partial x}, \tag{6}$$

Using the length and time rescaling [2]

$$X = \eta x, \qquad \tau = t - c^{-1} x \tag{7}$$

we rewrite the system of equations (1)-(6) in new variables

$$\frac{\partial(\alpha_i \rho_i)}{\partial \tau} + \eta \frac{\partial(\alpha_i \rho_i \upsilon_i)}{\partial X} - c^{-1} \frac{\partial(\alpha_i \rho_i \upsilon_i)}{\partial \tau} = 0, \tag{8}$$

$$\begin{aligned} &\frac{\partial(\alpha_i \rho_i \upsilon_i)}{\partial \tau} + \eta \frac{\partial(\alpha_i \rho_i \upsilon_i \upsilon_i)}{\partial X} - c^{-1} \frac{\partial(\alpha_i \rho_i \upsilon_i \upsilon_i)}{\partial \tau} = \\ &= \delta_{1i}\eta \frac{\partial \sigma}{\partial X} - c^{-1}\delta_{1i} \frac{\partial \sigma}{\partial \tau} + \eta\alpha_i \frac{\partial P}{\partial X} - c^{-1}\alpha_i \frac{\partial P}{\partial \tau} - (-1)^i R_{12}, \end{aligned} \tag{9}$$

$$\left\{b_0+\sum_{l=1}^{m}b_l\prod_{q=1}^{l}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial X}-c^{-1}\upsilon_1\frac{\partial}{\partial\tau}\right)^q\right\}(\sigma+\gamma P)=$$
$$=\left\{a_0+\sum_{l=1}^{n}a_l\prod_{q=1}^{l}\left(\frac{\partial}{\partial\tau}+\eta\upsilon_1\frac{\partial}{\partial X}-c^{-1}\upsilon_1\frac{\partial}{\partial\tau}\right)^q\right\}e, \tag{10}$$

$$\frac{\partial e_1}{\partial\tau}+\eta\frac{\partial e_1\upsilon_1}{\partial X}-c^{-1}\frac{\partial e_1\upsilon_1}{\partial\tau}=\eta\frac{\partial\upsilon_1}{\partial X}-c^{-1}\frac{\partial\upsilon_1}{\partial\tau}\quad, \tag{11}$$

We represent the desired variables in the form of series in small parameter $\eta<<1$

$$\begin{aligned}
&\alpha_1=\alpha_1^{(0)}+\eta\alpha_1^{(1)}+\eta^2\alpha_1^{(2)}+...;\\
&\alpha_2=\alpha_2^{(0)}+\eta\alpha_2^{(1)}+\eta^2\alpha_2^{(2)}+...;\\
&\rho_1=\rho_1^{(0)}+\eta(D_1\sigma_1+L_1P_1)+\\
&+\eta^2(D_1\sigma_2+L_1P_2+D_2\sigma_1^2+D_l\sigma_1P_1+L_2P_1^2)+...;\\
&\rho_2=\rho_2^{(0)}+\eta B_1P_1+\eta^2(B_1P_2+B_2P_1^2)+...;\\
&\sigma=\eta\sigma_1+\eta^2\sigma_2+...;P=\eta P_1+\eta^2P_2+...;\\
&\upsilon_i=\eta\upsilon_i^{(1)}+\eta^2\upsilon_i^{(2)}+...;
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&B_1=\left.\frac{\partial\rho_2}{\partial P}\right|_{P_0};B_2=\left.\frac{1}{2!}\frac{\partial^2\rho_2}{\partial P^2}\right|_{P_0};L_1=\left.\frac{\partial\rho_1}{\partial P}\right|_{P_0};D_1=\left.\frac{\partial\rho_1}{\partial\sigma}\right|_{\sigma_0};\\
&L_2=\left.\frac{1}{2!}\frac{\partial^2\rho_1}{\partial P^2}\right|_{P_0};D_2=\left.\frac{1}{2!}\frac{\partial^2\rho_1}{\partial\sigma^2}\right|_{\sigma_0};D_L=\left.\frac{\partial^2\rho_1}{\partial\sigma\partial P}\right|_{(P_0,\sigma_0)},
\end{aligned} \tag{13}$$

where $\alpha_1^{(0)},P_0$ and σ_0 are the values of concentration, pressure and effective stress in the fixed two-phase continuum $\upsilon_1^{(0)}=\upsilon_2^{(0)}=0$.

The substitution of expansions (12) into system of equations (8)-(11) and equaling the coefficients of members with the same degrees η at the first approximation is led to the system of homogeneous equations

$$\begin{aligned}
&\alpha_1^{(0)}D_1\sigma_1+\alpha_1^{(0)}L_1P_1+\rho_1^{(0)}\alpha_1^{(1)}-c^{-1}\alpha_1^{(0)}\rho_1^{(0)}\upsilon_1^{(1)}=0\\
&\rho_2^{(0)}\alpha_2^{(1)}+\alpha_2^{(1)}B_1P_1-c^{-1}\alpha_2^{(0)}\rho_2^{(0)}\upsilon_2^{(1)}=0\\
&\alpha_1^{(0)}\rho_1^{(0)}\upsilon_1^{(1)}+c^{-1}\sigma_1+c^{-1}\alpha_1^{(0)}P_1=0\\
&\alpha_2^{(0)}\rho_2^{(0)}\upsilon_2^{(1)}+c^{-1}\alpha_2^{(0)}P_1=0\\
&b_0(\sigma_1+\gamma P_1)=a_0e_1\ ,\ \alpha_1^{(1)}+\alpha_2^{(1)}=0\ ,\ e_1=-c^{-1}\upsilon_1^{(1)}.
\end{aligned} \tag{14}$$

The system (14) has a nontrivial solution if its determinant vanishes, that gives the following dispersion equation with respect to the velocity of linear waves c

$$\begin{aligned}
&\alpha_1^{(0)}\rho_1^{(0)}b_0[\alpha_1^{(0)}\rho_2^{(0)}(L_1-D_1\gamma)+\rho_1^{(0)}\alpha_2^{(0)}B_1]c^4+\\
&+[\alpha_1^{(0)}\rho_2^{(0)}\left(\alpha_1^{(0)}a_0D_1-a_0L_1+\alpha_1^{(0)}\rho_1^{(0)}b_0-\gamma\rho_1^{(0)}b_0\right)+\\
&+\alpha_2^{(0)}\rho_1^{(0)}\left(\alpha_1^{(0)}\rho_1^{(0)}b_0-a_0B_1\right)]c^2--\alpha_2^{(0)}\rho_1^{(0)}a_0=0\,.
\end{aligned}\tag{15}$$

Equation (15) has a pair of roots corresponding to propagation of longitudinal waves in solid and fluid phases [1].

From (14) subsequently we substitute the desired variables through the rate of the solid phase $\upsilon_1^{(1)}$ in which it is at the second approximation

$$\begin{gathered}
e_1=-c^{-1}\upsilon_1^{(1)},\\
P_1=-c\left(\alpha_1^{(0)}\rho_1^{(0)}-\frac{a_0}{b_0c^2}\right)\frac{\upsilon_1^{(1)}}{\alpha_1^{(0)}-\gamma},\;\;\upsilon_2^{(1)}=-\frac{P_1}{\rho_2^{(0)}c},\\
\sigma_1=c\alpha_1^{(0)}\rho_1^{(0)}\left(\gamma-\frac{a_0}{b_0c^2}\right)\frac{\upsilon_1^{(1)}}{\alpha_1^{(0)}-\gamma},\\
\alpha_2^{(1)}=-\alpha_1^{(1)}=c^{-1}\alpha_2^{(0)}\left(\alpha_1^{(0)}\rho_1^{(0)}-\frac{a_0}{b_0c^2}\right)\frac{1+c^2B_1}{\rho_2^{(0)}\left(\alpha_1^{(0)}-\gamma\right)}\upsilon_1^{(1)}
\end{gathered}\tag{16}$$

2. Quasi-uniform grid and approximation of initial boundary value problem

As we have already noted, the replacement of an infinite domain finite gives low accuracy. Therefore, here we will use a quasi-uniform grid [4]. Suppose that $T=T(\xi)$ a strictly monotone function defined on $[-1,\ 1]$, for which

$$\lim_{\xi\to-1}T(\xi)=-\infty\quad,\quad\lim_{\xi\to1}T(\xi)=+\infty$$

An example of such functions $T(\xi)=tg\left(\frac{\pi\xi}{2}\right)$ can be taken. Let's take the nodal points $\xi_n=\frac{n}{N}$, где $-N\le n\le N$, where N is a natural number, and n is a whole number. It is clear that $\xi_{-N}=-1,\ \xi_N=1$. Consider the grid

$$\omega_N=\left\{T(\xi_n),\ \xi_n=\frac{n}{N},\ -N\le n<N\right\}.$$

These are called a quasi-uniform grid in $(-\infty;\ +\infty)$

$$T_n=T(\xi_n)=tg\left(\frac{\pi\xi_n}{2}\right).$$

An example of such a network on the line is the so-called tangential grid with the nodal points

$$T_n = T(\xi_n) = tg\left(\frac{\pi\xi_n}{2}\right).$$

As in [3] a grid approximation of derivatives on an infinite field of quasi-uniform grid is given. In this case, for derivatives I and II order obtain

$$\left(\frac{\partial\upsilon}{\partial T}\right)_{n+\frac{1}{2}} \approx \frac{\upsilon_{n+1}-\upsilon_n}{T_{n+1}-T_n}\ , \qquad (17)$$

$$\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_n \approx \frac{2}{x_{n+1}-x_{n-1}}\left(\frac{\upsilon_{n+1}-\upsilon_n}{T_{n+1}-T_n}-\frac{\upsilon_n-\upsilon_{n-1}}{T_n-T_{n-1}}\right). \qquad (18)$$

And for the approximation of the derivative of the third order will have

$$\left(\frac{\partial^3\upsilon}{\partial T^3}\right)_{n+\frac{1}{2}} \approx \frac{1}{T_{n+1}-T_n}\left[\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_{n+1}-\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_n\right]. \qquad (19)$$

Note that these approximations when $n = N-1$ are unsuitable because the denominator we obtain in this case, to set correct boundary conditions is not possible. Therefore it is necessary to build an approximation, which does not contain a nodal point T_N, but has υ_N. To do this, instead of formulas (11)-(13) can be as in [3] to approximate fractional derivatives nodal points. Then, instead of (11) - (13) we have

$$\left(\frac{\partial\upsilon}{\partial T}\right)_{n+\frac{1}{2}} \approx \frac{\upsilon_{n+1}-\upsilon_n}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}, \qquad (20)$$

$$\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_n \approx \frac{1}{T_{n+\frac{1}{2}}-T_{n-\frac{1}{2}}}\left[\left(\frac{\partial\upsilon}{\partial T}\right)_{n+\frac{1}{2}}-\left(\frac{\partial\upsilon}{\partial T}\right)_{n-\frac{1}{2}}\right], \qquad (21)$$

$$\left(\frac{\partial^3\upsilon}{\partial T^3}\right)_{n+\frac{1}{2}} \approx \frac{1}{T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}}\left[\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_{n+1}-\left(\frac{\partial^2\upsilon}{\partial T^2}\right)_n\right], \qquad (22)$$

$$\left(\frac{\partial\upsilon}{\partial x}\right)_k \approx \frac{\upsilon^{k+1}-\upsilon^k}{x_{n+1}-x_n}=\frac{\upsilon^{k+1}-\upsilon^k}{l}\ , \qquad (23)$$

$$D_{l,h} = \left\{ \begin{matrix} (x_k, T_n) : x_k = k * l, T_n = tg\left(\dfrac{\pi \xi_n}{2}\right), \xi_n = \dfrac{n}{N}, \\ n = -N, \dots, -1, 0, 1, \dots, N \end{matrix} \right\}$$

$$\frac{\upsilon_n^{k+1} - \upsilon_n^k}{l} = \upsilon_n^k \cdot \left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{1}{2}}^k - R_1 \upsilon_n^k + B \upsilon_n^k \left|\upsilon_n^k\right| + R_2 A_2 \left(\frac{\partial^2 \upsilon}{\partial T^2}\right)_n^k - R_3 A_3 \left(\frac{\partial^3 \upsilon}{\partial T^3}\right)_{n+\frac{1}{2}}^k . \tag{24}$$

Here

$$\left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{1}{2}}^k \approx \frac{\upsilon_{n+1}^k - \upsilon_n^k}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)}, \quad \left(\overline{\frac{\partial \upsilon}{\partial T}}\right)_{n+\frac{1}{2}}^k \approx \frac{\upsilon_n^k - \upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}} - T_{n-\frac{3}{4}}\right)} \tag{25}$$

$$\left(\frac{\partial^2 \upsilon}{\partial T^2}\right)_n^k \approx \frac{1}{T_{n+\frac{1}{2}} - T_{n-\frac{1}{2}}} \left[\left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{1}{2}}^k - \left(\frac{\partial \upsilon}{\partial T}\right)_{n-\frac{1}{2}}^k \right] =$$

$$= \frac{1}{T_{n+\frac{1}{2}} - T_{n-\frac{1}{2}}} \left[\frac{\upsilon_{n+1}^k - \upsilon_n^k}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)} - \frac{\upsilon_n^k - \upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}} - T_{n-\frac{3}{4}}\right)} \right] \tag{26}$$

$$\left(\frac{\partial^3 \upsilon}{\partial T^3}\right)_{n+\frac{1}{2}}^k \approx \frac{1}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)} \left[\left(\frac{\partial^2 \upsilon}{\partial T^2}\right)_{n+1}^k - \left(\frac{\partial^2 \upsilon}{\partial T^2}\right)_n^k \right] =$$

$$= \frac{1}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)} \left\{ \frac{1}{T_{n+\frac{3}{2}} - T_{n+\frac{1}{2}}} \left[\left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{3}{2}}^k - \left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{1}{2}}^k \right] - \right.$$

$$\left. - \frac{1}{T_{n+\frac{1}{2}} - T_{n-\frac{1}{2}}} \left[\left(\frac{\partial \upsilon}{\partial T}\right)_{n+\frac{1}{2}}^k - \left(\frac{\partial \upsilon}{\partial T}\right)_{n-\frac{1}{2}}^k \right] \right\} =$$

$$= \frac{1}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)} \left\{ \frac{1}{T_{n+\frac{3}{2}} - T_{n+\frac{1}{2}}} \left[\frac{\upsilon_{n+2}^k - \upsilon_{n+1}^k}{2\left(T_{n+\frac{7}{4}} - T_{n+\frac{5}{4}}\right)} - \frac{\upsilon_{n+1}^k - \upsilon_n^k}{2\left(T_{n+\frac{3}{4}} - T_{n+\frac{1}{4}}\right)} \right] - , \right. \tag{27}$$

$$\left. - \frac{1}{T_{n+\frac{1}{2}} - T_{n-\frac{1}{2}}} \left[\frac{\upsilon_n^k - \upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}} - T_{n-\frac{3}{4}}\right)} - \frac{\upsilon_{n-1}^k - \upsilon_{n-2}^k}{2\left(T_{n-\frac{5}{4}} - T_{n-\frac{7}{4}}\right)} \right] \right\}$$

$$\frac{\upsilon_n^{k+1}-\upsilon_n^k}{l}=\upsilon_n^k\cdot\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}-R_1\upsilon_n^k+B\upsilon_n^k\left|\upsilon_n^k\right|+$$

$$+R_2A_2\frac{1}{T_{n+\frac{1}{2}}-T_{n-\frac{1}{2}}}\left[\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}-\frac{\upsilon_n^k-\upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}}-T_{n-\frac{3}{4}}\right)}\right]-$$

$$-R_3A_3\frac{1}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}\left\{\frac{1}{T_{n+\frac{3}{2}}-T_{n+\frac{1}{2}}}\left[\frac{\upsilon_{n+2}^k-\upsilon_{n+1}^k}{2\left(T_{n+\frac{7}{4}}-T_{n+\frac{5}{4}}\right)}-\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}\right]-\right.$$

$$\left.-\frac{1}{T_{n+\frac{1}{2}}-T_{n-\frac{1}{2}}}\left[\frac{\upsilon_n^k-\upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}}-T_{n-\frac{3}{4}}\right)}-\frac{\upsilon_{n-1}^k-\upsilon_{n-2}^k}{2\left(T_{n-\frac{5}{4}}-T_{n-\frac{7}{4}}\right)}\right]\right\}.$$

And so the following equation

$$\upsilon_n^{k+1}=\upsilon_n^k+l\upsilon_n^k\cdot\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}-lR_1\upsilon_n^k+lB\upsilon_n^k\left|\upsilon_n^k\right|+$$

$$+lR_2A_2\frac{1}{2\left(T_{n+\frac{1}{2}}-T_{n-\frac{1}{2}}\right)}\left[\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}-\frac{\upsilon_n^k-\upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}}-T_{n-\frac{3}{4}}\right)}\right]-$$

$$-lR_3A_3\frac{1}{T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}}\left\{\frac{1}{T_{n+\frac{3}{2}}-T_{n+\frac{1}{2}}}\left[\frac{\upsilon_{n+2}^k-\upsilon_{n+1}^k}{2\left(T_{n+\frac{7}{4}}-T_{n+\frac{5}{4}}\right)}-\frac{\upsilon_{n+1}^k-\upsilon_n^k}{2\left(T_{n+\frac{3}{4}}-T_{n+\frac{1}{4}}\right)}\right]-\right.$$

$$\left.-\frac{1}{T_{n+\frac{1}{2}}-T_{n-\frac{1}{2}}}\left[\frac{\upsilon_n^k-\upsilon_{n-1}^k}{2\left(T_{n-\frac{1}{4}}-T_{n-\frac{3}{4}}\right)}-\frac{\upsilon_{n-1}^k-\upsilon_{n-2}^k}{2\left(T_{n-\frac{5}{4}}-T_{n-\frac{7}{4}}\right)}\right]\right\},$$

$$-N+2\le n\le N-2,\quad k=0,1,\ldots \tag{28}$$

where

$$R_1 = \frac{\alpha_2^{(0)}(1+c^2B_1)}{c\rho_2^{(0)}(\alpha_1^{(0)}-\gamma)^2}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)\left(c^{-2}\rho_1^{(0)} - 2\rho_2^{(0)}L_1 + 2\rho_1^{(0)}B_1\right) +$$

$$+\alpha_1^{(0)}\rho_2^{(0)}\left(2\rho_1^{(0)}\gamma - \frac{3a_0}{b_0c^2}\right)D_1 - c^{-2}\alpha_1^{(0)}\rho_1^{(0)}\rho_2^{(0)} +$$

$$+\frac{a_0\alpha_1^{(0)}\rho_1^{(0)}\rho_2^{(0)}}{b_0c^3}\left(2\alpha_1^{(0)}D_1 + c^{-2}\right) + 2c^{-3}\alpha_1^{(0)}\rho_1^{(0)}\rho_2^{(0)}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right) +$$

$$+2c^{-3}\,\frac{\alpha_2^{(0)}\rho_1^{(0)}}{\rho_2^{(0)}\left(\alpha_1^{(0)}-\gamma\right)^2}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)^3 +$$

$$+\frac{2c\alpha_1^{(0)}\rho_2^{(0)}}{\left(\alpha_1^{(0)}-\gamma\right)^2}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)\left[\alpha_1^{(0)}\alpha_1^{(0)}\left(\rho_1^{(0)}\gamma - \frac{a_0}{b_0c^2}\right)^2 D_2 -\right.$$

$$-\alpha_1^{(0)}\left(\rho_1^{(0)}\gamma - \frac{a_0}{b_0c^2}\right)\times\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)D_L + \left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)^2 L_2 +$$

$$\left.+\frac{\alpha_2^{(0)}\rho_1^{(0)}}{\alpha_1^{(0)}\rho_2^{(0)}}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)^2 B_2\right]/\Delta\ ,$$

$$R_2 = K_\nu\rho_2^{(0)}\left[\frac{a_0}{b_0c^2}\left(\alpha_1^{(0)}D_1 + c^{-2}\rho_1^{(0)}/\rho_2^{(0)}\right) + c^{-2}\alpha_1^{(0)}\rho_1^{(0)}\left(1 - \frac{\rho_1^{(0)}}{\rho_2^{(0)}}\right)\times\right.$$

$$\left.\times\left(1 - \frac{\alpha_1^{(0)}\rho_1^{(0)} - a_0/\left(b_0c^2\right)}{\rho_1^{(0)}\left(\alpha_1^{(0)}-\gamma\right)}\right)\right]/\Delta$$

$$R_3 = \frac{\alpha_1^{(0)}\rho_1^{(0)}\rho_2^{(0)}}{b_0c^2}\left(\alpha_1^{(0)}D_1 + c^{-2}\right)/\Delta$$

$$\Delta = 2c\alpha_1^{(0)}\rho_1^{(0)}\rho_2^{(0)}\left[\alpha_1^{(0)}\left(\frac{a_0}{b_0c^2}D_1 + c^{-2}\rho_1^{(0)}\right) +\right.$$

$$\left.+\frac{c^{-2}\alpha_2^{(0)}}{\alpha_1^{(0)}\rho_2^{(0)}(\alpha_1^{(0)}-\gamma)}\left(\alpha_1^{(0)}\rho_1^{(0)} - \frac{a_0}{b_0c^2}\right)^2\right].$$

For sand medium we have

$$M = 5, L = 6, n = 88, m = 8, \rho_1^{(0)} = 262, \rho_2^{(0)} = 100,,$$
$$E_1 = 10^8, E_2 = 10^9, E_* = 10^7, \theta = 10^{-4}, \theta_* = 10^{-2},$$
$$\beta_1 = 3*10^{-10}, \beta_2 = 4{,}4*10^{-9}, M_1 = 10^7, M_2 = 10^6,$$
$$\alpha_1^{(0)} = 0.01, \alpha_2^{(0)} = 0.99, b_0 = 1, a_0 = 10^{-4}, \gamma = 0.2,$$

$$K_{\nu} = 5*10^6, L_1 = -\rho_1^{(0)} * \beta_1 / 3, D_1 = -\rho_1^{(0)} * \beta_1 / (3*(1-\alpha_1^{(0)})),$$
$$B_1 = -\rho_2^{(0)} * \beta_2, B_2 = 0, D_L = 0, L_2 = 0, D_2 = 0,$$
$$A_2 = -E_1 * \theta - E_* * \theta_*, A_3 = -(E_1 + E_*) * \theta * \theta_* - M_2,$$
$$h = 2*M/n, l = L/m, T_1 = -M, T_{n+1} = M, X_1 = 0, X_{m+1} = L$$

which was considered in [5,6].

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